

# Nonlinear surface magneto-optics of Fe/Cu(001) from first principles: Influences of the Cu substrate

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## Acknowledgements

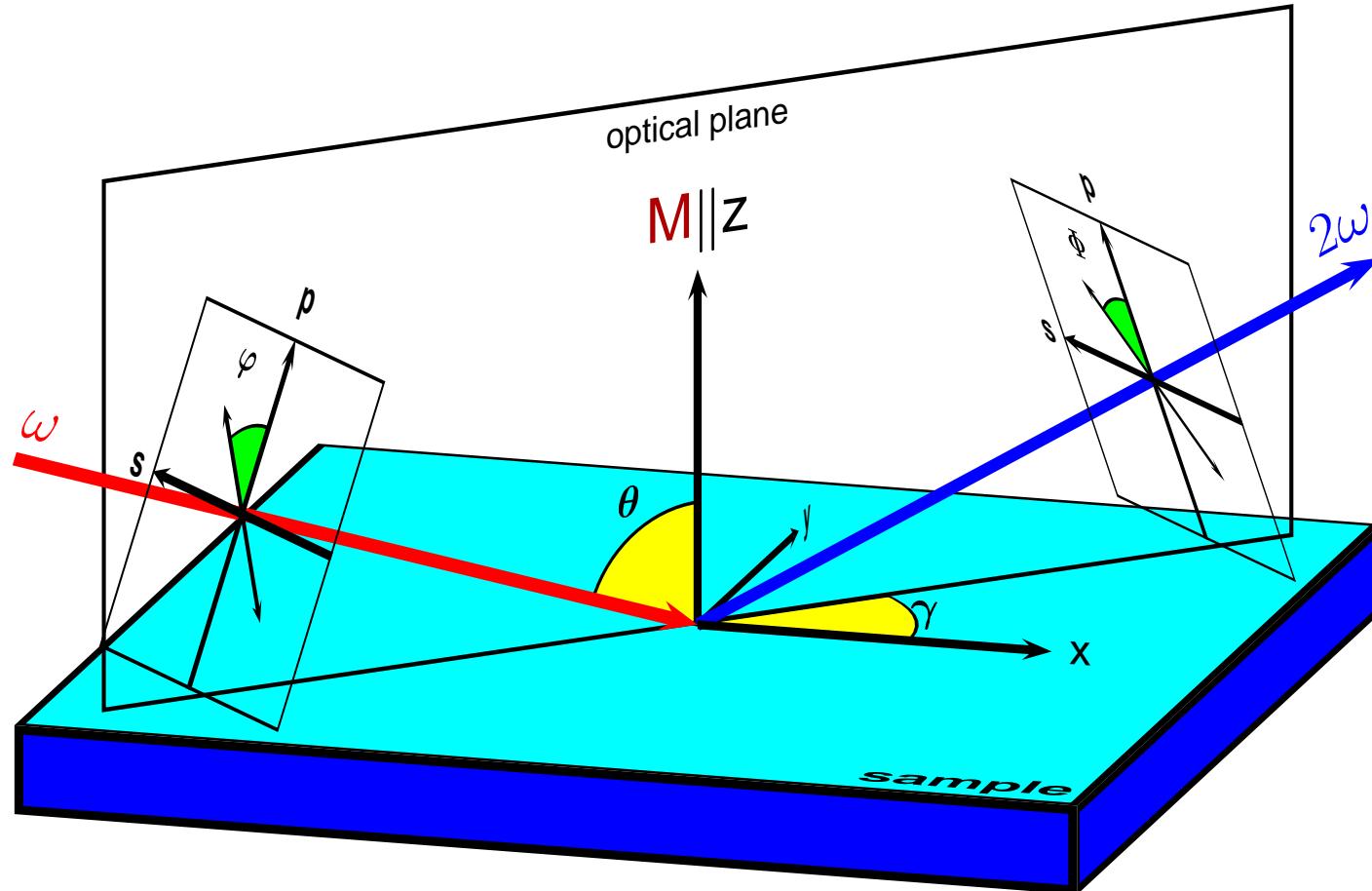
- Vector optimization: Rechenzentrum Garching
- Financial support: DFG and EU TMR “NOMOKE” and “DYNASPIN”

## Outline

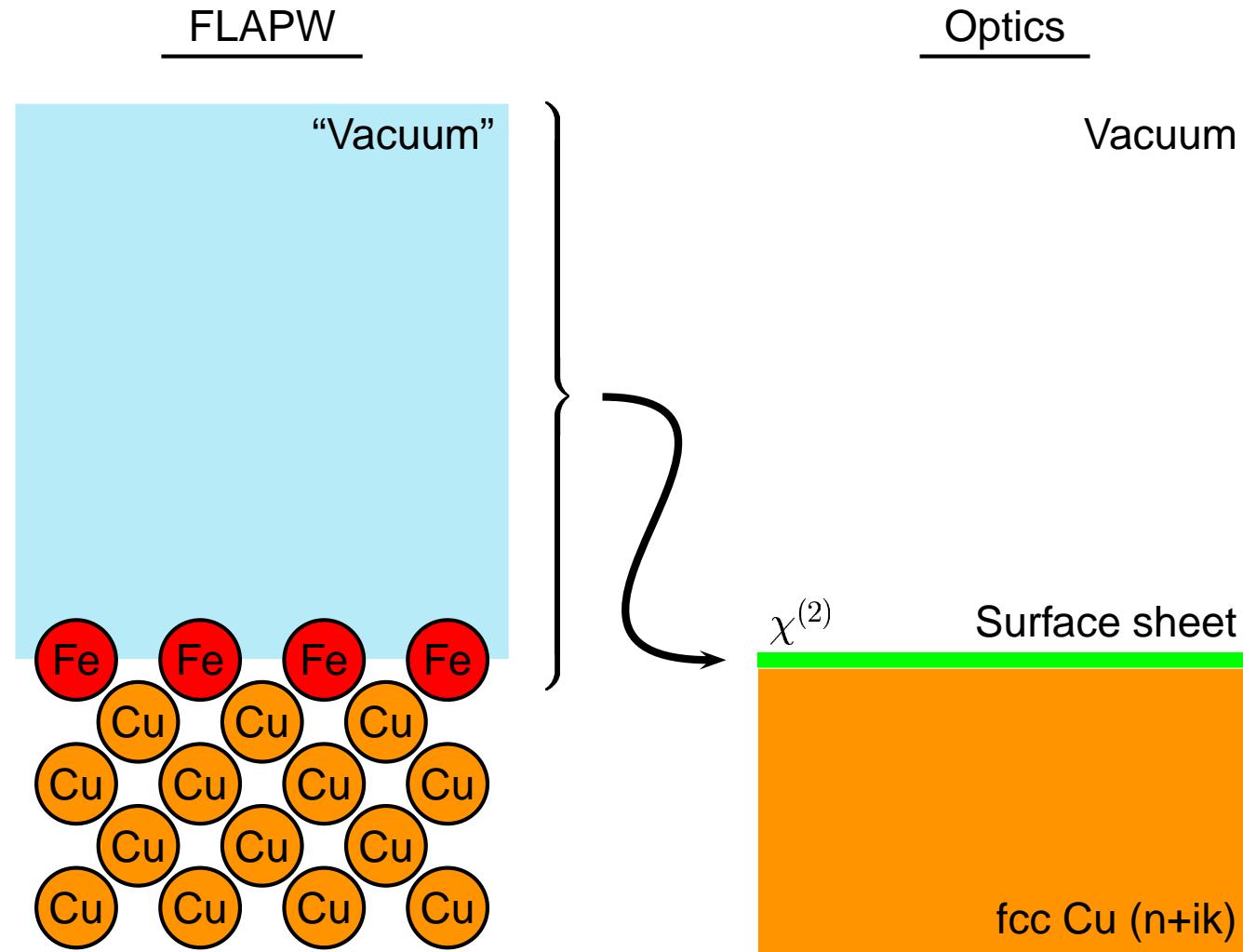
- Reflection geometry for nonlinear optics
- Numerical calculations: ▶ method ▶ formulas
- Results: ▶ band structure ▶ layer-resolved magnetic moments ▶ screened nonlinear susceptibility ▶ nonlinear optical intensities ▶ rotating the polarizer ▶ nonlinear Kerr rotation
- Conclusions and outlook



# Reflection geometry for SHG on magnetic surface



## Nonlinear magneto-optics – method



# Nonlinear optical field

$$E(2\omega; \theta, \Phi, \varphi, \gamma) = 2i\delta z \frac{\omega}{c} |E_0(\omega)|^2$$

$$\chi_{ijk}^{(2)}(2\mathbf{q}, 2\omega), \mathbf{M} \parallel \mathbf{z}$$

$$\times \left\{ A_s \sin \Phi \begin{pmatrix} -\sin \gamma \\ \cos \gamma \\ 0 \end{pmatrix} + A_p \cos \Phi \begin{pmatrix} F_c \cos \gamma \\ F_c \sin \gamma \\ N^2 F_s \end{pmatrix} \right\} \cdot \boxed{\begin{pmatrix} 0 & 0 & 0 & \textcolor{red}{xyz^-} & xxz^+ & 0 \\ 0 & 0 & 0 & xxz^+ & \textcolor{red}{-xyz^-} & 0 \\ zxx^+ & zxz^+ & zzz^+ & 0 & 0 & 0 \end{pmatrix}}$$

$$\cdot \begin{pmatrix} [t_p f_c \cos \varphi \cos \gamma - t_s \sin \varphi \sin \gamma]^2 \\ [t_p f_c \cos \varphi \sin \gamma + t_s \sin \varphi \cos \gamma]^2 \\ t_p^2 f_s^2 \cos^2 \varphi \\ 2[t_p f_c \cos \varphi \sin \gamma + t_s \sin \varphi \cos \gamma] t_p f_s \cos \varphi \\ 2[t_p f_c \cos \varphi \cos \gamma - t_s \sin \varphi \sin \gamma] t_p f_s \cos \varphi \\ 2[t_p f_c \cos \varphi \cos \gamma - t_s \sin \varphi \sin \gamma][t_p f_c \cos \varphi \sin \gamma + t_s \sin \varphi \cos \gamma] \end{pmatrix}$$

4

# Screened nonlinear optical susceptibility

$$\chi_{ijk}^{(2)}(2\mathbf{q}, 2\omega) = e^3 \sum_{\mathbf{k}, l, l', l''} \left\{ \frac{\langle \mathbf{k} + 2\mathbf{q}, l'' | \mathbf{r}_i | \mathbf{k}, l \rangle \langle \mathbf{k}, l | \mathbf{r}_j | \mathbf{k} + \mathbf{q}, l' \rangle \langle \mathbf{k} + \mathbf{q}, l' | \mathbf{r}_k | \mathbf{k} + 2\mathbf{q}, l'' \rangle}{E_{\mathbf{k}+2\mathbf{q},l''} - E_{\mathbf{k},l} - 2\hbar\omega + 2i\hbar\alpha} \right.$$

$$\times \left( \frac{f(E_{\mathbf{k}+2\mathbf{q},l''}) - f(E_{\mathbf{k}+\mathbf{q},l'})}{E_{\mathbf{k}+2\mathbf{q},l''} - E_{\mathbf{k}+\mathbf{q},l'} - \hbar\omega + i\hbar\alpha} - \frac{f(E_{\mathbf{k}+\mathbf{q},l'}) - f(E_{\mathbf{k},l})}{E_{\mathbf{k}+\mathbf{q},l'} - E_{\mathbf{k},l} - \hbar\omega + i\hbar\alpha} \right) \Big\}$$

$$\times \left[ 1 + 4\pi e^2 \sum_{ab} m_a m_b \sum_{\mathbf{k}, l, l''} \langle \mathbf{k}, l | \mathbf{r}_a | \mathbf{k} + 2\mathbf{q}, l'' \rangle \langle \mathbf{k} + 2\mathbf{q}, l'' | \mathbf{r}_b | \mathbf{k}, l \rangle \frac{f(E_{\mathbf{k}+2\mathbf{q},l''}) - f(E_{\mathbf{k},l})}{E_{\mathbf{k}+2\mathbf{q},l''} - E_{\mathbf{k},l} - 2\hbar\omega + 2i\hbar\alpha} \right]^{-1}$$

# Nonlinear intensity

$$I(2\omega; \theta, \Phi, \varphi, \gamma) = \epsilon_0 c_0 |E(2\omega; \theta, \Phi, \varphi, \gamma)|^2$$

# Details of the calculation

- WIEN97 spin-polarized scf cycle
  - energy convergence to 0.1 mRy
  - no inversion symmetry (surface)
- add magnetization direction
- our implementation of the spin-orbit coupling
  - second variation
  - spherical potential, inside MT sphere only
  - WIEN97 **basis set**
  - added after complete scf-cycle
- transition matrix elements  $\langle \mathbf{k}, l | \mathbf{r}_i | \mathbf{k}', l' \rangle$
- screened nonlinear susceptibility  $\chi_{ijk}^{(2)}$
- optical properties ( $E, I, \mathcal{A}, \phi_K$ )

## Parameters in WIEN97 scf cycle

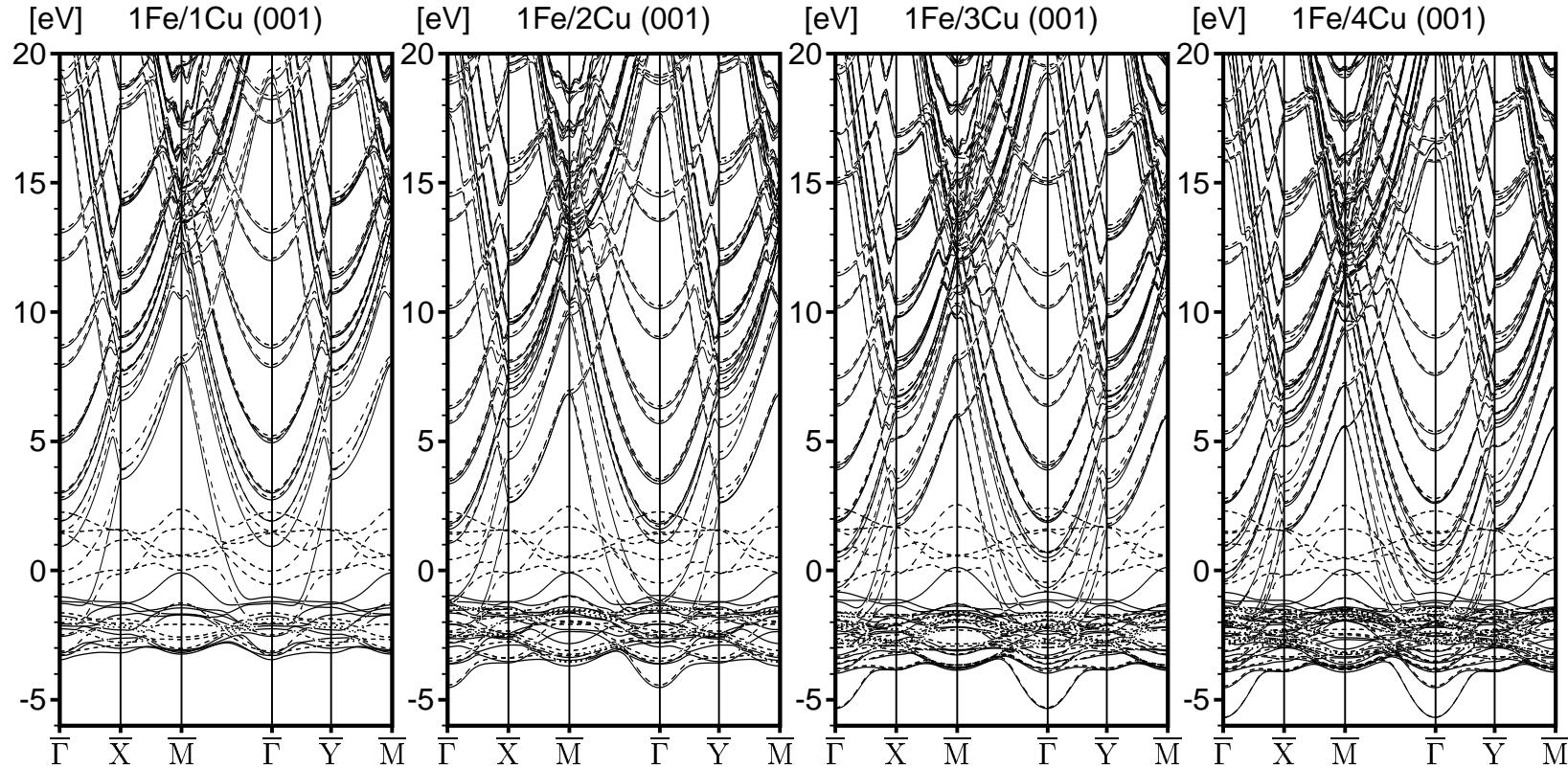
- fcc Cu lattice,  $a = 3.61 \text{ \AA}$
- separation 8 vacuum layers
- muffin-tin radius  $1.21 \text{ \AA}$
- 2502  $\mathbf{k}$ -points in I2BZ

## Parameters in optics

- $|E_0(\omega)| = 10^8 \text{ V/m}$
- $\delta z = 4.82 \text{ \AA}$
- $\gamma = 0, \theta = \pi/4$
- linear Cu  $n + ik$  from [1]



# Fe/Cu (001) Band structure with spin-orbit coupling



Solid lines: > 90% majority spin  
Dashed lines: > 90% minority spin  
Dotted lines: < 90% of either spin

## Magnetic spin moments

1 ML Fe on up to 7 ML fcc Cu (001)

Fe	Cu layer number						
	1	2	3	4	5	6	7
2.828	0.046						
2.844	0.043	-0.026					
2.793	0.033	-0.025	-0.003				
2.792	0.048	-0.015	-0.006	-0.065			
2.853	0.042	-0.016	-0.000	0.001	0.001		
2.835	0.044	-0.016	-0.002	-0.001	-0.002	-0.006	
2.740	0.045	-0.016	0.004	0.005	0.008	0.001	-0.010

FM AFM

First Cu layer FM coupled to Fe layer

Second Cu layer AFM coupled to first Cu layer

# Nonlinear optical field

$$E(2\omega; \theta, \Phi, \varphi, \gamma) = 2i\delta z \frac{\omega}{c} |E_0(\omega)|^2$$

$\chi_{ijk}^{(2)}(2\mathbf{q}, 2\omega), \mathbf{M} \parallel \mathbf{z}$

$$\times \left\{ A_s \sin \Phi \begin{pmatrix} -\sin \gamma \\ \cos \gamma \\ 0 \end{pmatrix} + A_p \cos \Phi \begin{pmatrix} F_c \cos \gamma \\ F_c \sin \gamma \\ N^2 F_s \end{pmatrix} \right\} \cdot \boxed{\begin{pmatrix} 0 & 0 & 0 & \textcolor{red}{xyz^-} & xxz^+ & 0 \\ 0 & 0 & 0 & xxz^+ & \textcolor{red}{-xyz^-} & 0 \\ zxx^+ & zxz^+ & zzz^+ & 0 & 0 & 0 \end{pmatrix}}$$

$$\cdot \begin{pmatrix} [t_p f_c \cos \varphi \cos \gamma - t_s \sin \varphi \sin \gamma]^2 \\ [t_p f_c \cos \varphi \sin \gamma + t_s \sin \varphi \cos \gamma]^2 \\ t_p^2 f_s^2 \cos^2 \varphi \\ 2[t_p f_c \cos \varphi \sin \gamma + t_s \sin \varphi \cos \gamma] t_p f_s \cos \varphi \\ 2[t_p f_c \cos \varphi \cos \gamma - t_s \sin \varphi \sin \gamma] t_p f_s \cos \varphi \\ 2[t_p f_c \cos \varphi \cos \gamma - t_s \sin \varphi \sin \gamma][t_p f_c \cos \varphi \sin \gamma + t_s \sin \varphi \cos \gamma] \end{pmatrix}$$

8

# Screened nonlinear optical susceptibility

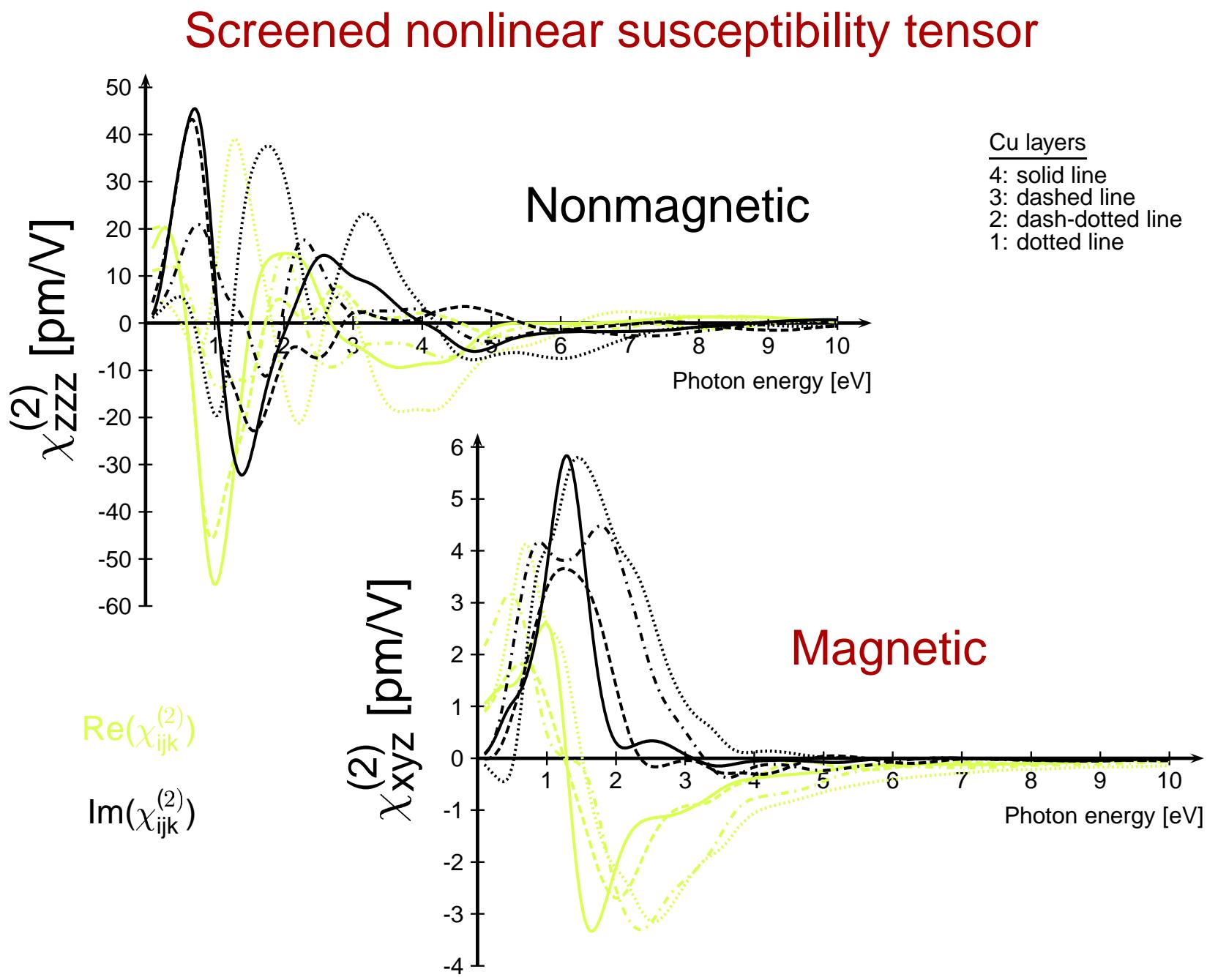
$$\chi_{ijk}^{(2)}(2\mathbf{q}, 2\omega) = e^3 \sum_{\mathbf{k}, l, l', l''} \left\{ \frac{\langle \mathbf{k} + 2\mathbf{q}, l'' | \mathbf{r}_i | \mathbf{k}, l \rangle \langle \mathbf{k}, l | \mathbf{r}_j | \mathbf{k} + \mathbf{q}, l' \rangle \langle \mathbf{k} + \mathbf{q}, l' | \mathbf{r}_k | \mathbf{k} + 2\mathbf{q}, l'' \rangle}{E_{\mathbf{k}+2\mathbf{q},l''} - E_{\mathbf{k},l} - 2\hbar\omega + 2i\hbar\alpha} \right.$$

$$\times \left( \frac{f(E_{\mathbf{k}+2\mathbf{q},l''}) - f(E_{\mathbf{k}+\mathbf{q},l'})}{E_{\mathbf{k}+2\mathbf{q},l''} - E_{\mathbf{k}+\mathbf{q},l'} - \hbar\omega + i\hbar\alpha} - \frac{f(E_{\mathbf{k}+\mathbf{q},l'}) - f(E_{\mathbf{k},l})}{E_{\mathbf{k}+\mathbf{q},l'} - E_{\mathbf{k},l} - \hbar\omega + i\hbar\alpha} \right) \Big\}$$

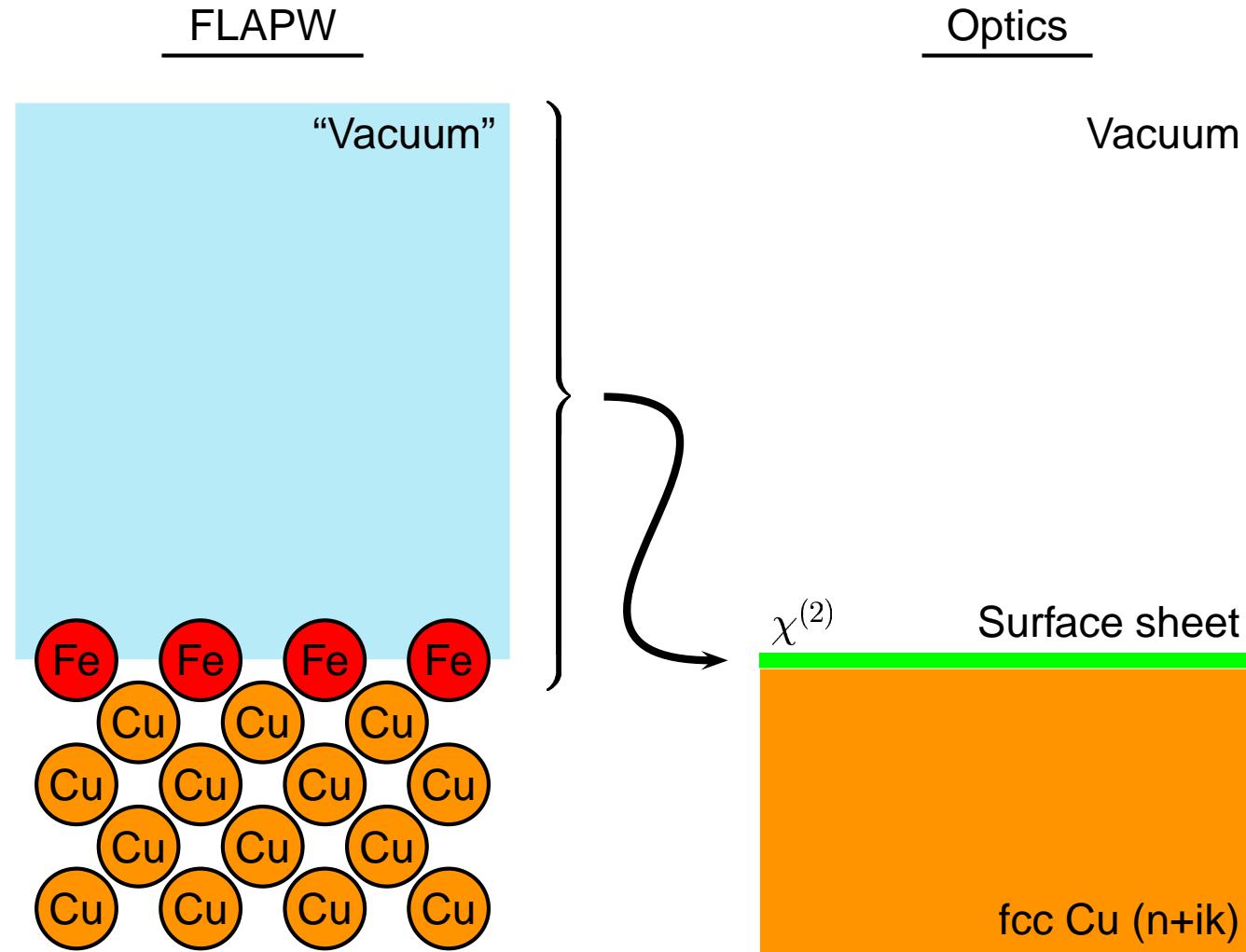
$$\times \left[ 1 + 4\pi e^2 \sum_{ab} m_a m_b \sum_{\mathbf{k}, l, l''} \langle \mathbf{k}, l | \mathbf{r}_a | \mathbf{k} + 2\mathbf{q}, l'' \rangle \langle \mathbf{k} + 2\mathbf{q}, l'' | \mathbf{r}_b | \mathbf{k}, l \rangle \frac{f(E_{\mathbf{k}+2\mathbf{q},l''}) - f(E_{\mathbf{k},l})}{E_{\mathbf{k}+2\mathbf{q},l''} - E_{\mathbf{k},l} - 2\hbar\omega + 2i\hbar\alpha} \right]^{-1}$$

# Nonlinear intensity

$$I(2\omega; \theta, \Phi, \varphi, \gamma) = \epsilon_0 c_0 |E(2\omega; \theta, \Phi, \varphi, \gamma)|^2$$

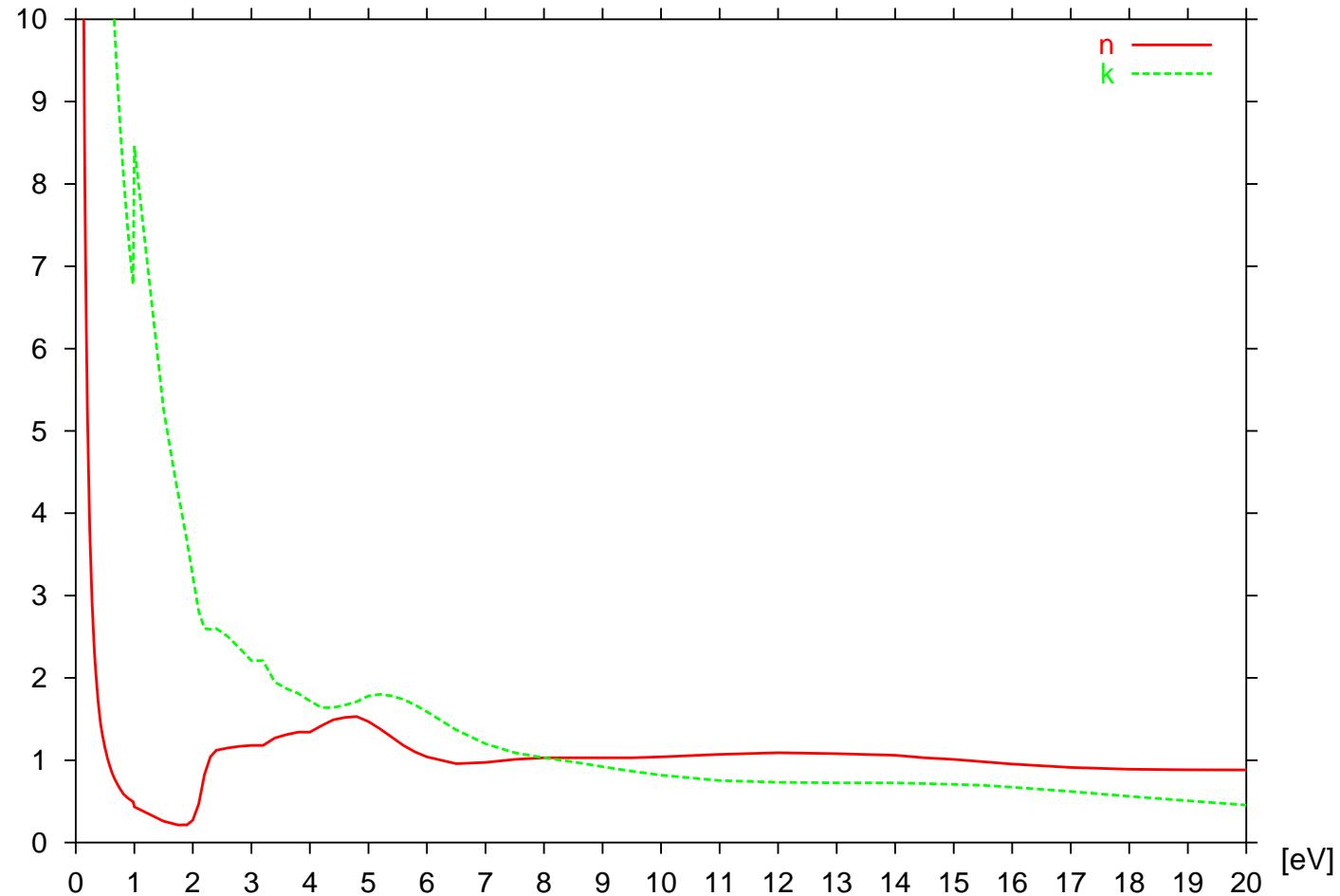


## Nonlinear magneto-optics – method



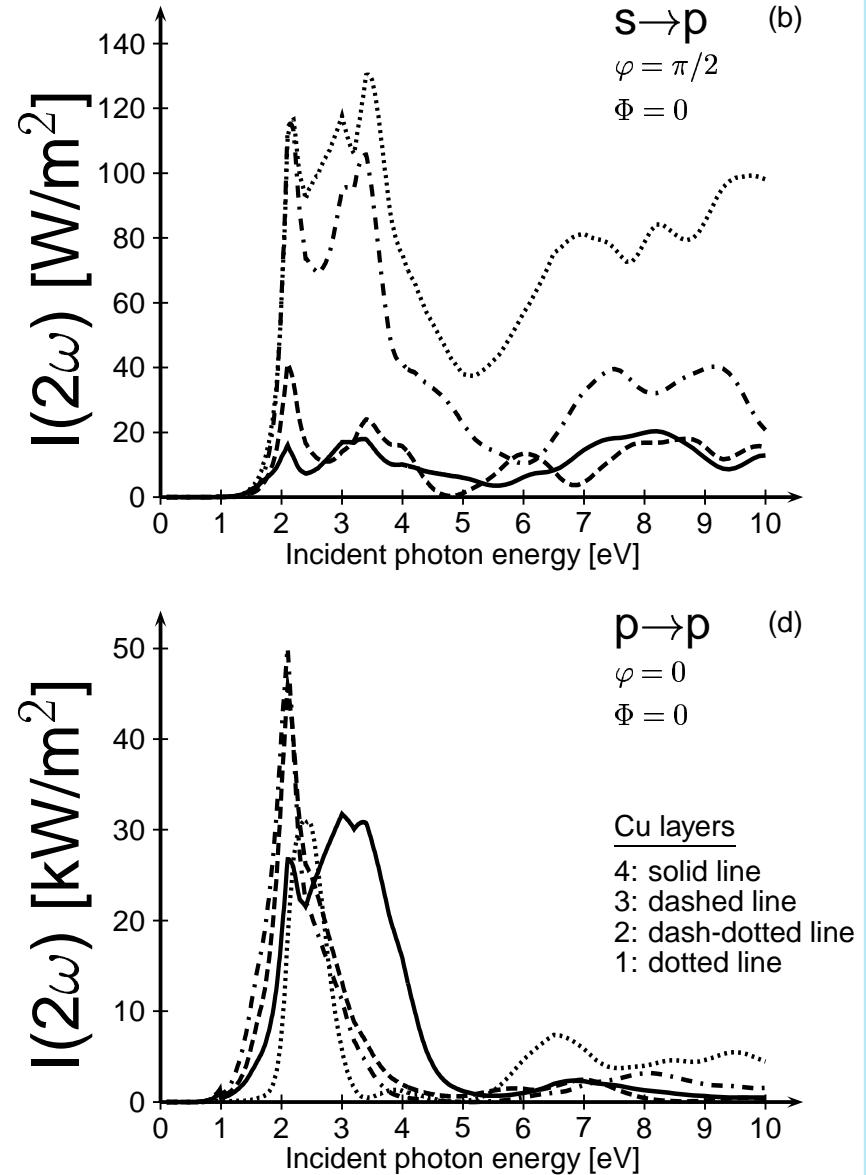
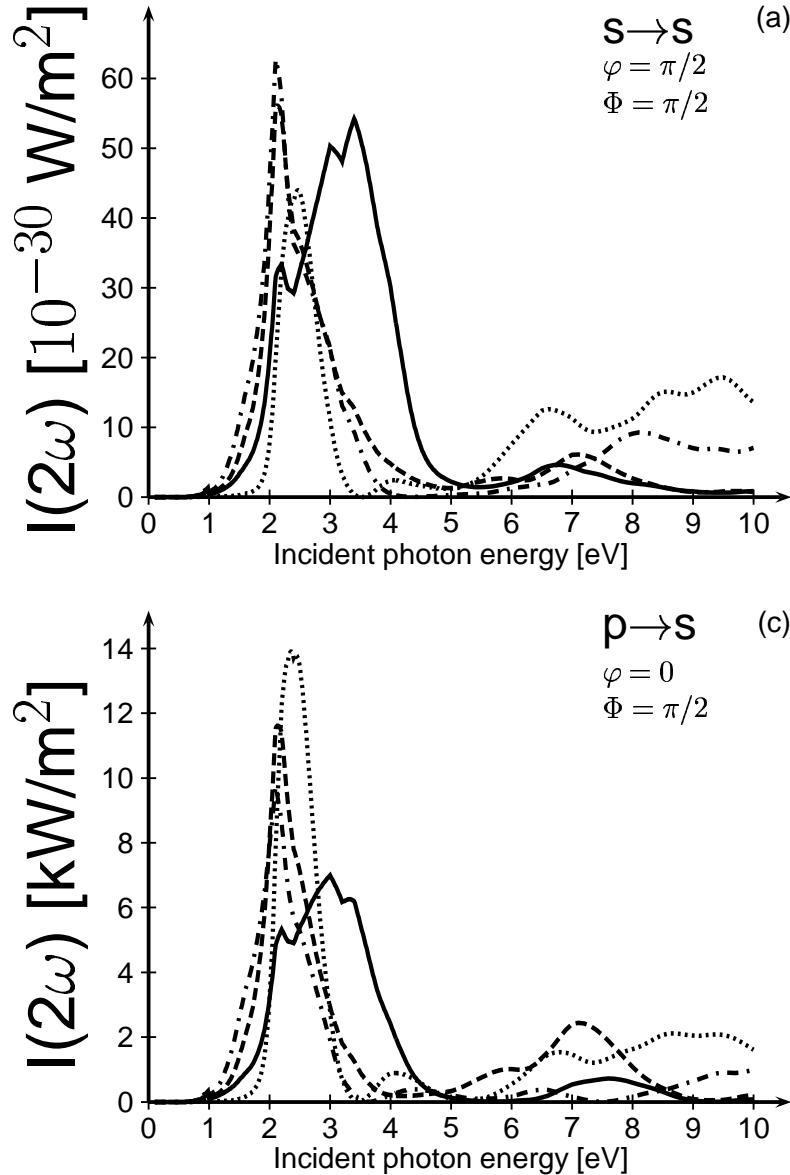
# Complex refractive index ( $n+ik$ ) of Cu

## Experimental values



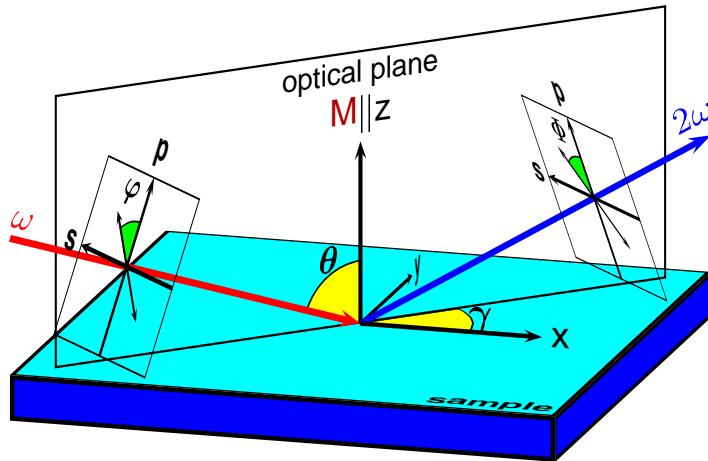
[D. W. Lynch and W. R. Hunter, in *Handbook of optical constants in solids*, edited by E. D. Palik (Academic Press, London, 1985), pp. 275–367]

## Second-harmonic intensities



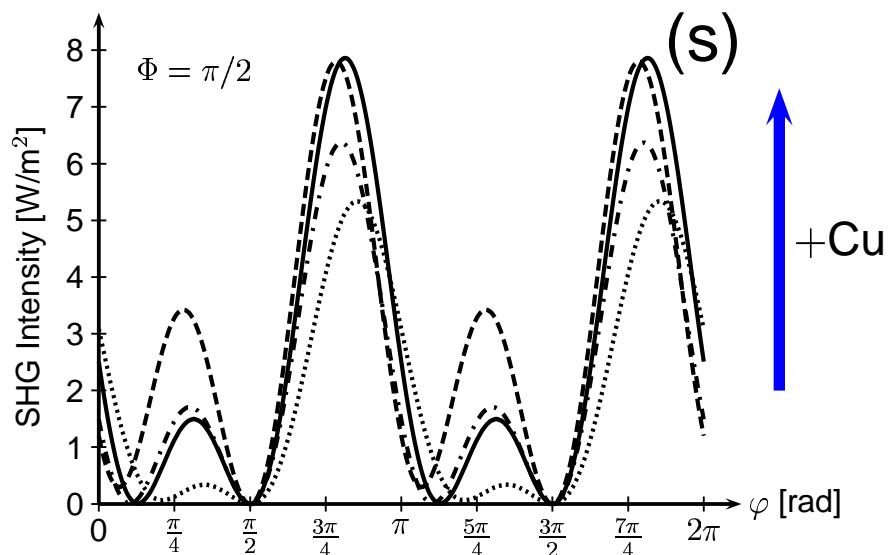
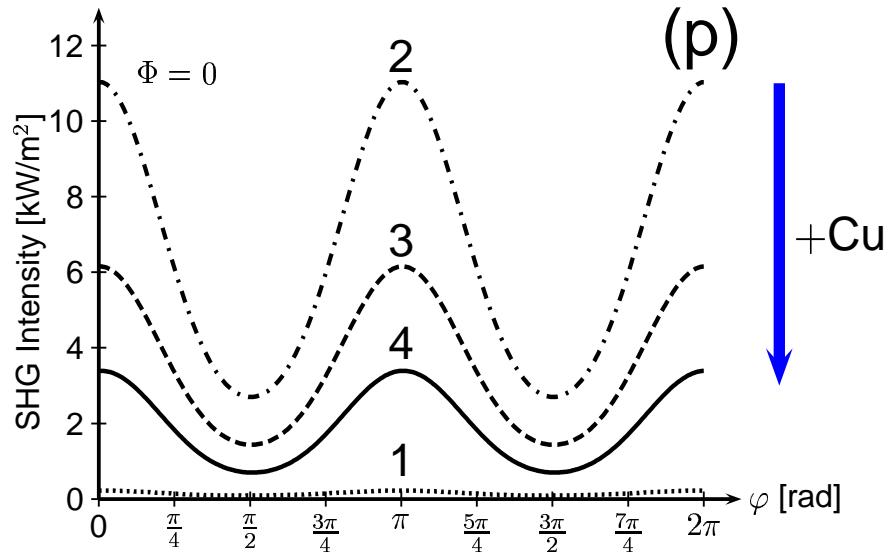
## Varying the polarizer angle $\varphi$

$\hbar\omega = 1.5 \text{ eV}$



### Cu layers

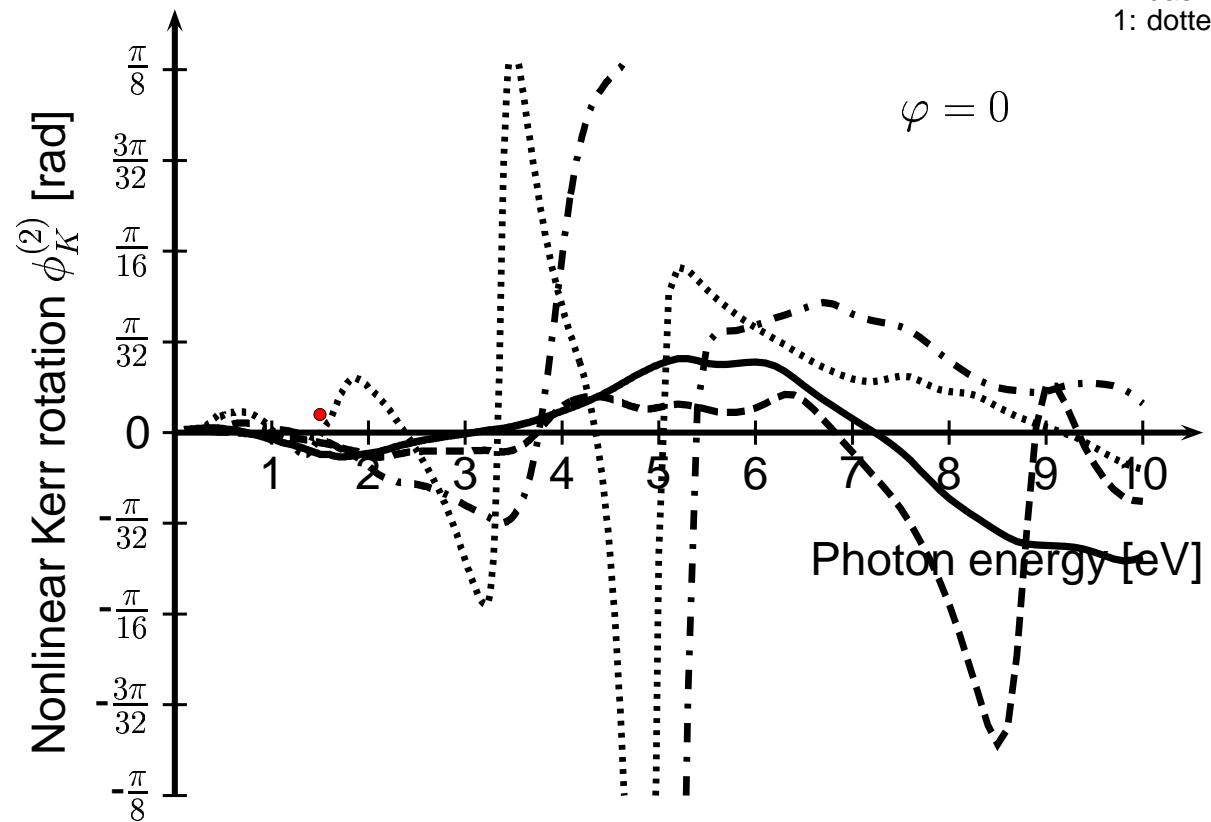
- 4: solid line
- 3: dashed line
- 2: dash-dotted line
- 1: dotted line



## Nonlinear ( $2\omega$ ) magneto-optical Kerr rotation

$$\phi^{(2)} = \frac{1}{2} \arctan \frac{2\text{Re}[E_s(2\omega)/E_p(2\omega)]}{1 - |E_s(2\omega)/E_p(2\omega)|^2} + \phi_0$$

Cu layers  
4: solid line  
3: dashed line  
2: dash-dotted line  
1: dotted line



## Conclusions

- large magnetic tensor element – powerful tool
- adding Cu gives another degree of freedom
- nonlinear optics can be calculated using first principles methods (WIEN97)
- symmetry breaking by magnetism can be resolved
- susceptibilities for 3 and 4 Cu underlayers close
- intensities do not look that converged for 4 Cu layers
- Kerr rotation not converged

Further details in: Torsten Andersen and W. Hübner, *Substrate effects in magneto-optical second-harmonic generation from first principles: Fe/Cu(001)*, to appear in Phys. Rev. B.

## Outlook

- Wien2k spin-orbit?
- correct occupied d-band and excited state positions – GW
- nonspherical contributions to spin-orbit interaction
- nonlocality ( $|\mathbf{q}| \neq 0$ )
- removal of translational invariance along  $z$

# Nonlocal theory of nonlinear magneto-optics

## Mesoscopic systems

- Finite size
- Surfaces
- Interfaces
- Short light pulses

## Magnetism

- Spin-orbit coupling
- Relativistic electrons

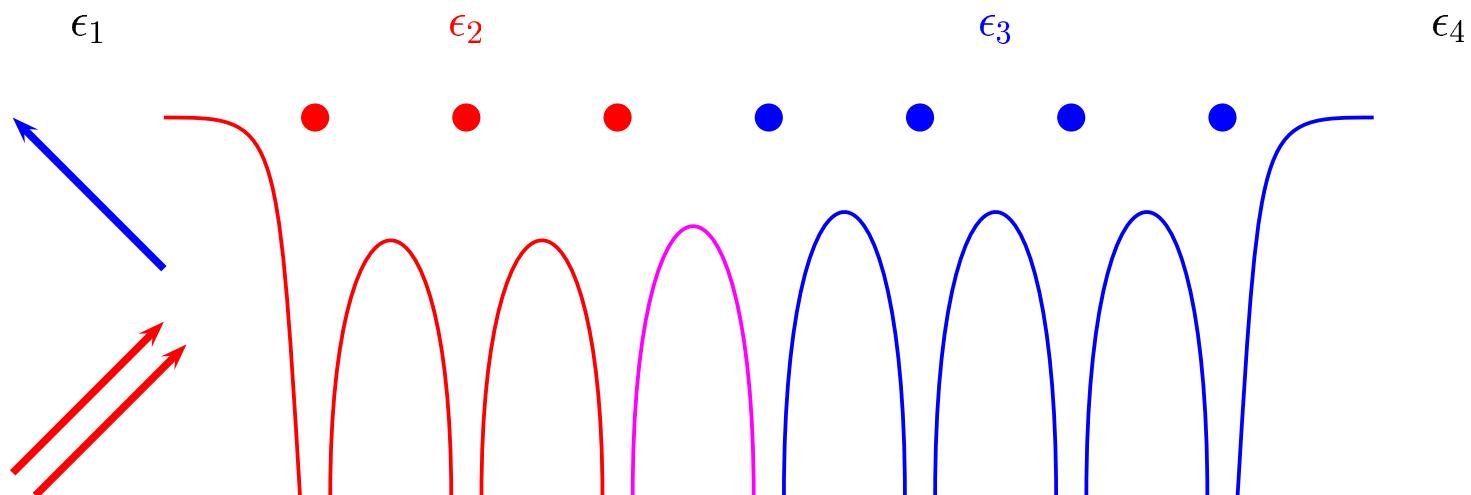
## Nonlocality

- Delocalized  $s$ -electrons
- Relativistic  $d$ -electrons

## Problems

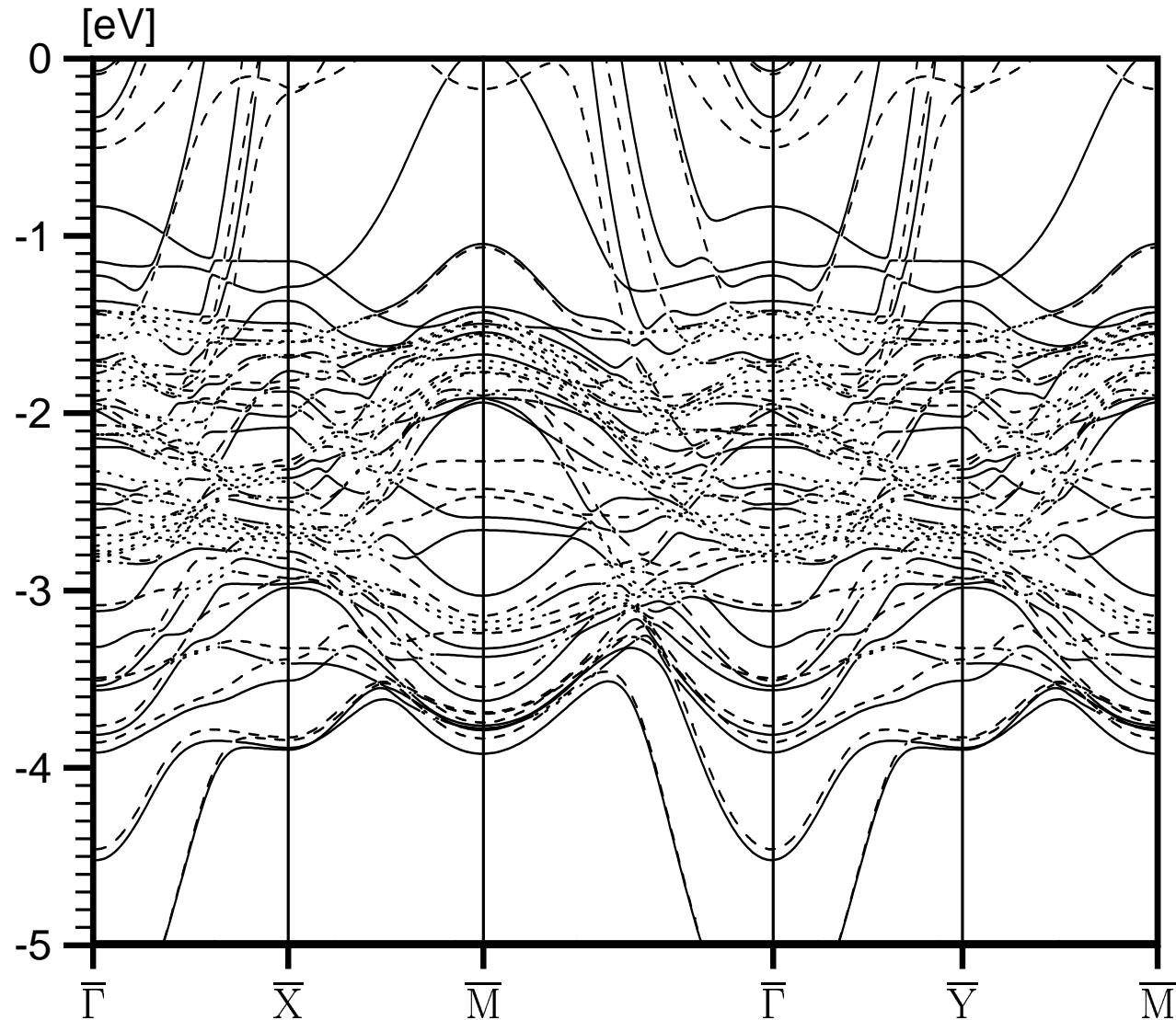
- Boundaries?
- Large field gradients  $\Rightarrow \cancel{\text{ED}}$
- ED + EQ + MD + ⋯ ?

## Nonlinear optics

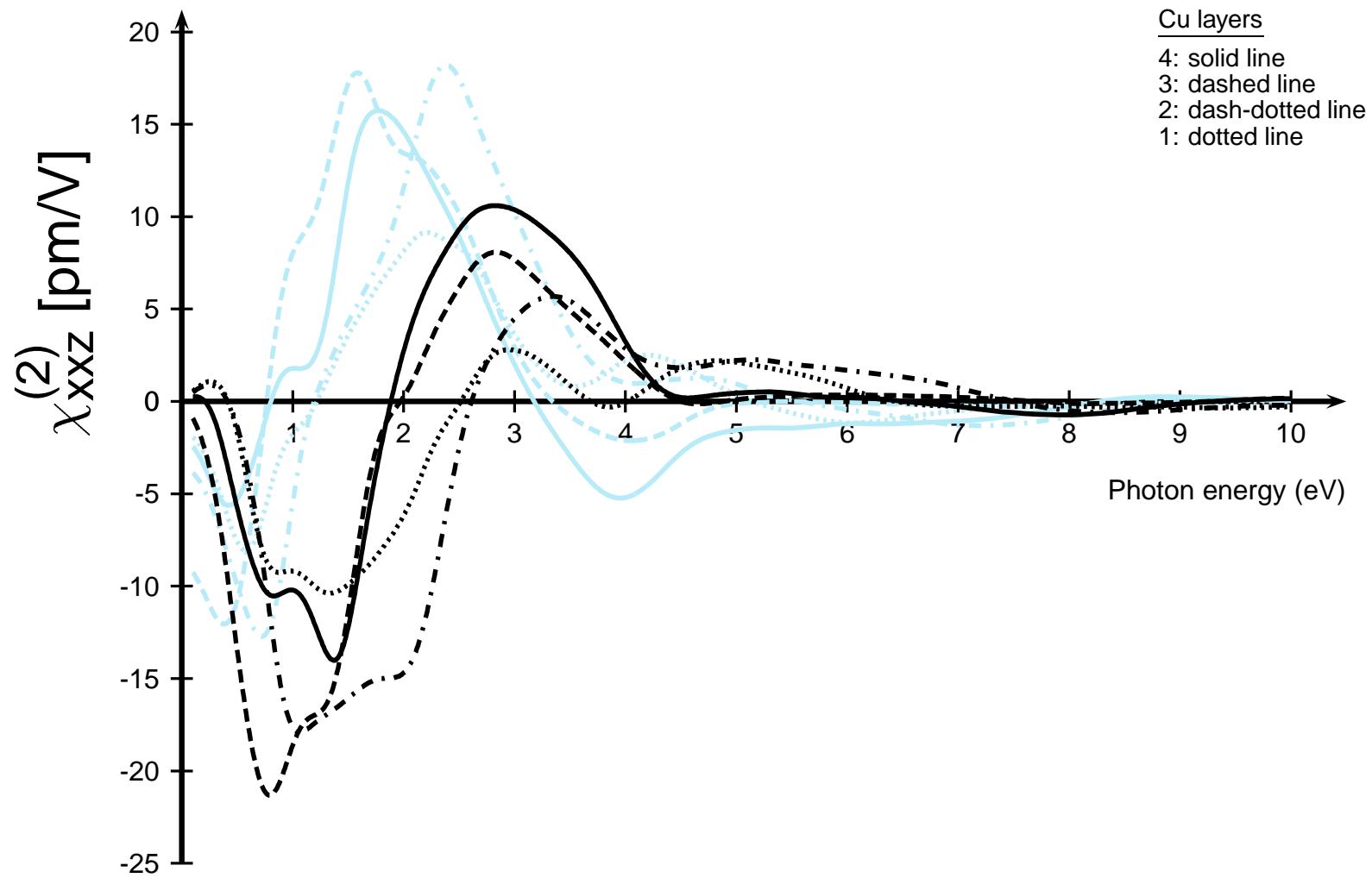


# Band structure with spin-orbit coupling

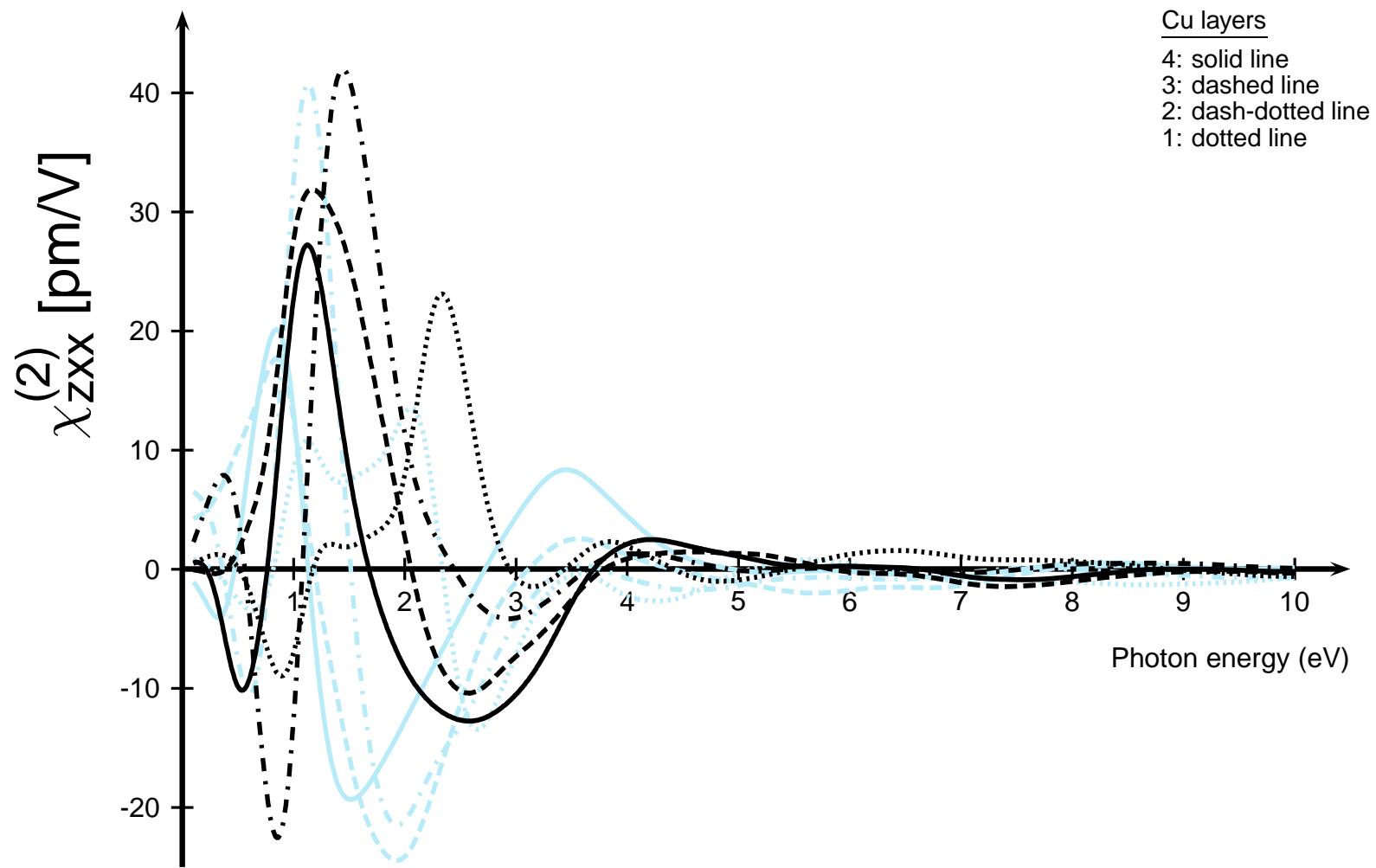
1 ML Fe on 4 ML Cu (001)



## Screened nonlinear susceptibility tensor



## Screened nonlinear susceptibility tensor



# Magneto-optical asymmetry $\mathcal{A}$ as function of $\varphi$

$$\mathcal{A} = \frac{I(\mathbf{M}\|\mathbf{z}) - I(\mathbf{M}\|-\mathbf{z})}{I(\mathbf{M}\|\mathbf{z}) + I(\mathbf{M}\|-\mathbf{z})}$$

$\hbar\omega=1.5$  eV

Cu layers  
 4: solid line  
 3: dashed line  
 2: dash-dotted line  
 1: dotted line

